Robust INS/SFPPP-BDS tightly coupled navigation algorithm with adaptive cycle slip compensation model

Hangshuai Ma¹,², Rong Wang³, Zhi Xiong³, Jianye Liu³ and Chuanyi Li³

Abstract
The application of Beidou Satellite Navigation System (BDS) is developing rapidly. To satisfy the increasing demand for positioning performance, single-frequency precise point positioning (SFPPP) has been a focus in recent years. By introducing the SFPPP technique into the INS/BDS integrated system, higher navigation accuracy can be obtained. Cycle slip, which is caused by signal blockage during the measurement of the carrier phase, is a challenge for SFPPP application. In the INS/SFPPP-BDS integrated system, cycle slip can cause serious bias in BDS carrier phase measurements. In this paper, a new INS/SFBDs-PPP tightly coupled navigation system and a robust adaptive filtering method are proposed. Using a low-cost single-frequency receiver integrated with INS, an observation model was built based on the pseudo range and carrier phase by PPP preprocessing. The cycle slip was introduced into the state vector to improve the estimation precision. The test statistics, comprising the innovation and its covariance, were used to estimate the time at which cycle slip occurred and its amplitude to compensate for its effect on the observation. Finally, the proposed system model and algorithm are validated by simulation.

Keywords
Integrated navigation, tightly coupled INS/BDS integration, single-frequency precise point positioning, cycle slip compensation model, robust fusion

Received 30 June 2018; Revised received 5 October 2018; accepted 28 October 2018

Introduction
With the development of Beidou Satellite Navigation System (BDS), the INS/GNSS integrated navigation technology has been applied in aerospace, surveying, and vehicle positioning.¹-³ To satisfy the increasing demand for positioning performance, single-frequency precise point positioning (SFPPP) has been a focus in recent years.⁴,⁵ Unlike the traditional BDS positioning relay in the pseudo range and with only a pseudo range rate, the SFPPP introduces carrier phase measurements and information on the precise ephemeris and clock bias from the International GNSS Service (IGS), which reduce the measurement noise level and improve the positioning accuracy.⁶ Thus, by introducing the SFPPP technique into the INS/BDS integrated system, higher navigation accuracy can be obtained.⁷-⁹

Cycle slip, which is caused by signal blockage during the measurement of the carrier phase, is a challenge for SFPPP application. In the INS/SFPPP-BDS integrated system,¹⁰ cycle slip can cause serious bias in BDS carrier phase measurements. If carrier phase measurements with cycle slip were to be directly used in the fusion filter of the INS/SFPPP-BDS integrated system, the estimation would be incorrect, resulting in
divergence of navigation results.\textsuperscript{11–14} Since signal blockage frequently occurs, particularly during the attitude maneuvering of aircraft or the operation of vehicles in an urban environment, the robustness of the INS/SFPPP-BDS to cycle slip must be improved.

In recent years, many methods have been studied to reduce the effect of cycle slip. To apply SFPPP to the tightly coupled INS/BDS system, the key point is to solve the problem caused by the cycle slip of the carrier phase. In INS/SFPPP-BDS, cycle slip introduces inaccuracies into the estimated INS errors in the system filter and thereby to the navigation results. Meanwhile, cycle slip detection and repair have become a major research focus. Hidekazu Machidam developed a remedy for phase error caused by cycle slip, with S-shaped drive pattern for smooth acceleration and thereby to the navigation results. Meanwhile, cycle slip detection and repair have become a major research focus. Hidekazu Machidam developed a remedy for phase error caused by cycle slip, with S-shaped drive pattern for smooth acceleration and thereby to the navigation results.

Nejad et al.\textsuperscript{17} proposed a method to fix cycle slips using multi-constellation and multi-frequency GNSS data, which uses the average code and phase linear combination and applies to static data.\textsuperscript{16} Zangeneh-Nejad et al.\textsuperscript{17} proposed a method to fix cycle slips based on the generalized likelihood ratio (GLR) test. Teunissen studied the strength of the single-receiver, single-channel GNSS model for instantaneously resolving integer cycle-slips, including multi-frequency GPS, Galileo and BDS, thereby focusing on the challenging case that the slip is due to a simultaneous loss of lock on all frequencies.\textsuperscript{18} Scholz\textsuperscript{19} analyzed the performance for detecting and correcting cycle slips in carrier phase measurements in short signal outages situation in urban environments. Yousef et al.\textsuperscript{20} developed a method to determine the change in the ionospheric and the code bias values from epoch to epoch by solving all observable equations using the least square technique, which could detect and predict cycle slip size of 1 cycle. Henkel and Oku\textsuperscript{21} researched a cycle slips search method for double difference receivers and verified in both a slalom drive with high vehicle dynamics and a drive below trees with shadowed GPS signals. Liu\textsuperscript{22} studied a cycle slip detection and repair method for a single dual-frequency GPS receiver; Cai et al.\textsuperscript{23} studied cycle slip detection and repair for GPS without differential observations under high ionospheric activity; Tao studied the cycle slip detection and repair of Beidou;\textsuperscript{24} and Huang et al.\textsuperscript{25} studied a cycle slip algorithm validated with GPS data. These methods can detect and repair cycle slip to a certain degree.

However, for INS/SFPPP-BDS, the system filter works by continuous feedback correction, and the bias of the cycle slip repair will accumulate the effects of the INS error and cause navigation errors. Therefore, cycle slip and the integrated navigation system filter are important topics to address for INS/SFPPP-BDS.

In this paper, a new INS/SFPPP-BDS tightly coupled navigation system and a robust adaptive filtering method are proposed. Using a low-cost single-frequency receiver integrated with INS, an observation model was built based on the pseudo range and carrier phase by PPP preprocessing. The cycle slip was introduced into the state vector to improve the estimation precision. The test statistics, comprising the innovation and its covariance, were used to estimate the time at which cycle slip occurred and its amplitude to compensate for its effect on the observation. Finally, the proposed system model and algorithm are validated by simulation.

**Observation model of SF BDS-PPP**

**General model**

The observation vector of INS/SFPPP-BDS is composed of two parts: a pseudo-range observation, which is the difference between the code pseudo-range measurements from a BDS receiver and an equivalent pseudo-range of INS, and a half-sum observation, which is derived from the code pseudo-range and carrier phase.

The error of the pseudo-range and carrier phase measurements can be decreased by the IGS products. The clock error is eliminated by the IGS precise ephemeris; the ionospheric error can be decreased by the grid layer model;\textsuperscript{26,27} the wet delay of the troposphere is estimated as a state of the system filter. The pseudo-range and carrier phase measurements of the BDS are shown in equations (1) and (2)

\[
P_{G_i} = \rho_i - Cdt + M_w \cdot d_w + d_{ion} + v_{p_i} \quad (1)
\]

\[
\phi_i = \rho_i - Cdt + M_w \cdot d_w - d_{ion} - \lambda \cdot N_i + v_{\phi_i} \quad (2)
\]

where \(P_{G_i}\) is the pseudo-range measurement of the \(i_{th}\) visible satellite; \(\phi_i\) is the carrier phase measurement of the \(i_{th}\) visible satellite; \(\rho_i\) is the real geometric distance between the BDS receiver and the \(i_{th}\) visible satellite; \(dt\) is the clock error of the \(i_{th}\) visible satellite; \(d_w\) is the troposphere delay; \(M_w\) is the wet component delay function; \(d_{ion}\) is the ionosphere delay; \(\lambda\) is the wavelength of the carrier phase measurement; \(N_i\) is the integer ambiguity of the \(i_{th}\) visible satellite; \(v_{p_i}\) and \(v_{\phi_i}\) are the observation errors of the pseudo-range and carrier phase, respectively; and \(C\) is the speed of light in vacuum.
**Improved model under the cycle slip effect**

The half-sum model is used in the single-frequency PPP because it can eliminate the ionosphere delay. The ionosphere delay has opposite effects on the pseudo-range and carrier phase, as shown in equations (1) and (2), so the half-sum model can eliminate the ionosphere delay, as shown in equation (3)

\[
D_{\text{half-sum}} = \frac{P_{G_i} + \varphi_i}{2} = \frac{\rho_i - Cdt + M_i \cdot d_i - \lambda_i N_i + \frac{v_{P_i} + v_{\Phi_i}}{2}}{C0}
\]

where \(D_{\text{half-sum}}\) is the half-sum measurement, which has no ionosphere delay. Assuming that the cycle slip occurs at epoch \(k\), the integer ambiguity will undergo an abrupt change, and the observation model will change as shown in equation (4)

\[
\hat{\varphi}_i = \varphi_i + \lambda \cdot \Delta N_i
\]

where \(\hat{\varphi}_i\) is the carrier phase measurement of the \(i_{th}\) visible satellite; \(\varphi_i\) is the real value of the carrier phase of the \(i_{th}\) visible satellite; \(\Delta N_i\) is the change in the integer ambiguity caused by the cycle slip, with \(\Delta N_i = 0\) when there is no cycle slip. The cycle slip does not affect the pseudo-range measurement because the carrier phase measurement is not used, but the half-sum measurement changes with the cycle slip, as shown in equation (5)

\[
\hat{D}_{\text{half-sum}} = \frac{P_{G_i} + \hat{\varphi}_i}{2} = \frac{\rho_i - Cdt + M_i \cdot d_i - \lambda_i N_i + \lambda \cdot \Delta N_i + \frac{v_{P_i} + v_{\Phi_i}}{2}}{C0}
\]

where \(\hat{D}_{\text{half-sum}}\) is the half-sum measurement with cycle slip in the carrier phase.

**Cycle slip compensation method**

If there is no cycle slip, the innovation follows white noise, and the mean innovation is zero; when there is cycle slip, the mean innovation changes with the cycle slip. We define the hypothesis testing as follows: when no observation has faults, the hypothesis is \(H_0\) (zero hypothesis); when at least one observation has a fault, the hypothesis is \(H_1\) (alternative hypothesis), as follows

\[
H_0: V_k \sim N(0, S_k) \\
H_1: V_k \sim N(\Delta N_k, S_k)
\]

where \(S_k\) is the innovation variance matrix at epoch \(k\), which can be derived from equation (7)

\[
S_k = H_k P_{k,k-1} H_k^T + R_k
\]

where \(H_k\) is the measurement matrix; \(P_{k,k-1}\) and \(R_k\) are the covariance matrices of the system states and system observations, respectively, which are both obtained from the initial setting of the system filtering.

We use the innovation to construct the test statistics of the cycle slip detection

\[
T_k = V_k^T S_k V_k
\]

According to the characters of normal distribution and the matrix operation rules, under both the zero hypothesis and the alternative hypothesis, \(T_k\) follows a chi-square distribution \(\chi^2\)

\[
H_0: T_k \sim \chi^2(r, 0) \\
H_1: T_k \sim \chi^2(r, \lambda_k)
\]

where \(r\) is the number of degrees of freedom of the distribution and \(\lambda_k\) is the noncentral parameter. Based on the given significance level, the judgment threshold \(T_R\) can be determined. When the test statistics are larger than \(T_R\), cycle slip should occur. Using the least-squares method, the optimal estimation of the cycle slip can be obtained, as shown in equation (10)

\[
\Delta N = S_k^{-1} V_k S_k
\]

**Robust INS/SFBDS-PPP integrated method**

**System scheme design**

The scheme of the INS/SFPPP-BDS tightly coupled navigation system is shown in Figure 1. In the proposed INS/SFPPP-BDS integrated navigation system, the INS is integrated with the SFPPP-BDS in a tightly coupled mode. The INS provides the attitude, velocity, and position results by the strapdown algorithm; the SFPPP-BDS provides the code pseudo-range and single-frequency carrier phase measurements. The IGS products such as the precise ephemeris and clock correction are synchronously received. This information will be transmitted to the system filter to construct the state model and observation model, based on which the state error estimation can be obtained to correct the INS and BDS in the feedback process.
The system observations consist of two parts: the pseudo-range observation, which is derived from the INS calculated pseudo-range and pseudo-range measurement from the BDS receiver, and the iono-free combination observation, which is derived from the half sum of the phase and pseudo-range. The system states also consist of two parts: INS errors and SFPPP-BDS positioning errors, which include receiver clock error, clock drift, tropospheric error, wet delay, integer ambiguity, and cycle slip.

**Modeling for the INS/SFPPP-BDS integrated system**

**INS error state model.** The system state model is given in equation (11)

\[
\dot{X} = FX + W
\]

where \( F \) is the state transition matrix; \( W \) is the system noise matrix; and \( X \) is the state vector. \( X \) is composed of INS states and SFPPP-BDS errors as follows

\[
X = [X_{INS} \quad X_{BDS}]^T = [\phi, v, p, \varepsilon_0, \varepsilon_r, b_o, d, dt, d_\theta, N, \Delta N]^T
\]

where \( \phi \) is the platform error angle vector; \( v \) and \( p \) are the velocity error vector and position error vector in geographical coordinates, respectively; \( \varepsilon_0 \) and \( \varepsilon_r \) are the random constant error vector and first-order Markov error vector of the gyros, respectively; \( b_o \) is the first-order Markov error vector of the accelerators; \( dt \) and \( d\theta \) are the clock error and clock drift, respectively; \( N \) is the integer ambiguity vector of the visible satellites; and \( \Delta N \) is the cycle slip of the carrier phase of the visible satellites.

The cycle slip state is modeled as a random constant, as shown in equation (13)

\[
\Delta \tilde{N}_i = 0
\]

where \( F_{INS} \) is the state transition matrix of the INS error model, as given in Groves, and \( F_{BDS} \) is the state transition matrix of the BDS states.

**INS/SFPPP-BDS measurement model.** The INS calculated pseudo-range can be obtained as follows

\[
P_i = \sqrt{(x_i - x_{i,j})^2 + (y_i - y_{i,j})^2 + (z_i - z_{i,j})^2}
\]

where \((x_i, y_i, z_i)\) is the position derived from INS; \((x_{i,j}, y_{i,j}, z_{i,j})\) is the position of the \(i_{th}\) visible satellite; and \(P_i\) is the equivalent pseudo-range between the position provided by INS and the \(i_{th}\) visible satellite.

The observation model of INS/SFPPP-BDS is composed of a pseudo-range observation part and a half-
sum observation part. The pseudo-range observation is the difference between the pseudo-range measurement of the BDS and the equivalent pseudo-range measurement of the INS; the half-sum observation is the linearization equation derived from the difference between the half-sum model and the equivalent pseudo-range measurement of the INS. The observation model is

\[
P_i = P_L - P_{Bi} = e_1\delta x + e_2\delta y + e_3\delta z + Cdt \]
\[L_i = P_L - D_{half-sum} = e_1\delta x + e_2\delta y + e_3\delta z \]

where \( P_i \) is the pseudo-range observation of INS/SFBDSSPPP; \( L_i \) is the half-sum observation of INS/SFBDSSPPP; and \( e_1, e_2, e_3 \) is the direction cosine of the receiver and satellites. The observation model can be written as

\[
Z = HX
\]

where \( X \) is the system state vector, as shown in equation (12); \( H \) is the system observation matrix derived from equation (16); \( Z \) is the system observation vector of INS/SFPPP-BDS, as shown in equation (18)

\[
Z = [P_1 \ P_2 \ \cdots \ P_n \ L_1 \ L_2 \ \cdots \ L_m]
\]

where \( n \) is the number of visible satellites.

The observation of the INS/SFPPP-BDS tightly coupled system changes as follows

\[
\hat{Z}_k = Z_k + \Delta Z_k
\]

where

\[
\Delta Z_k = \begin{bmatrix}
0, 0, \cdots, 0 \\
\frac{-\lambda \Delta N_1}{2}, -\frac{\lambda \Delta N_2}{2}, \cdots, -\frac{\lambda \Delta N_{svnum}}{2}
\end{bmatrix}
\]

where \( svnum \) is the number of visible satellites.

The innovation of the Kalman filter is defined as equation (21)

\[
V_k = \hat{Z}_k - Z_{k-1} = \hat{Z}_k - H_k X_{k,k-1}
\]

where \( V_k \) is the innovation of the filter; \( \hat{Z}_k \) is the current observation of the INS/SFPS-PPP tightly coupled system at epoch \( k \); \( Z_{k-1} \) is the observation forecast by the system filter; and \( H_k \) is the observation equation of the system filter. Equation (21) shows that the innovation includes both the current and the last observation information. Therefore, an abrupt change in the observation will be reflected in the innovation, which is useful for cycle slip detection.

### Robust fusion algorithm

\[
\Delta Z_k = \begin{bmatrix}
0, 0, \cdots, 0 \\
\frac{-\lambda \Delta N_1}{2}, -\frac{\lambda \Delta N_2}{2}, \cdots, -\frac{\lambda \Delta N_{svnum}}{2}
\end{bmatrix}
\]

Assuming that a cycle slip occurs at epoch \( k \), we use the optimal estimation to repair the observation with faults

\[
\hat{Z}_k = \begin{cases} 
\hat{Z}_k - \Delta \hat{Z}_k & \text{when } T_k > T_R \\
\hat{Z}_k & \text{when } T_k > T_R
\end{cases}
\]

For matrix \( P \), when there is no cycle slip, \( P \) converges with time; when cycle slip occurs, if the recursion of \( P \) retains the value at the last time, the system filter will be unstable and divergent, and the accuracy of the navigation results will decrease. When cycle slip occurs, the value of the cycle slip state \( \Delta N_k \) should be identical to the real value of the cycle slip, and the corresponding value of \( P \) should be identical. Therefore, we use equation (10) to reassign the value of state \( \Delta N_k \) in \( P \)

\[
P_{k,k-1}(i,i) = \begin{cases} 
\frac{\lambda}{2} \Delta N_i & \text{P matrix of cycle slip state} \\
P_{k,k-1}(i,i) & \text{P matrix of other states}
\end{cases}
\]

Then, the adaptive filtering of INS/SFPPP-BDS follows as equations (25) to (29).

\[
X_{k,k-1} = F_k X_{k-1}
\]

\[
X_k = F_k X_{k,k-1} + K_k (\hat{Z}_k - H_k X_{k,k-1})
\]

\[
K_k = \hat{P}_{k,k-1}^{-1} H_k^T (H_k \hat{P}_{k,k-1} H_k^T + R_k)^{-1}
\]

\[
P_k = (I - K_k H_k) \hat{P}_{k,k-1}
\]

\[
P_{k,k-1} = \begin{cases} 
\Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^T & \text{no cycle slip happens} \\
\hat{P}_{k,k-1}(i,i) & \text{cycle slip happen}
\end{cases}
\]
Simulation and analysis

In this section, simulation and analysis have been conducted to test the proposed scheme and the robust INS/SFPPP-BDS tightly coupled navigation algorithm with an adaptive cycle slip compensation model. The flight track in the simulation is set as a dynamic track, which includes climb up, cruise, and sharp turn. The simulation parameter settings are given in Table 1.

Simulation of the INS/SFPPP-BDS tightly coupled system

In this section, the positioning accuracy performance of the proposed INS/SFPPP-BDS tightly coupled integrated navigation system is tested in the simulation. Compared with the traditional INS/BDS tightly coupled system, the INS/SFPPP-BDS system uses more accurate pseudo-range and half-sum observations by partially eliminating the BDS clock error, ionospheric error and tropospheric error. Therefore, the INS/SFPPP-BDS system has better positioning precision than the INS/BDS tightly coupled system. The navigation error comparisons of the INS/BDS and INS/SFPPP-BDS systems under identical simulation conditions are shown in Figures 2 to 4.

In Figure 2, the dashed line represents the positioning errors of the INS/SFPPP-BDS tightly coupled system, and the full line represents the INS/BDS tightly coupled system. The positioning error of the INS/SFPPP-BDS system is smaller than 4 m, and the positioning error of the INS/BDS system is approximately 10 m. The main cause of the higher precision is the improved accuracy of the observations in the INS/SFPPP-BDS system. In the traditional INS/BDS system, the pseudo-range with the noise of 20–30 m, so the positioning error noise is large. In the INS/SFPPP-BDS system, the pseudo-range observations are smoothed by the carrier phase in PPP preprocessing, and the errors of the half-sum observation are corrected by the IGS products. The positioning errors of the two systems are compared in Table 2.

Table 1. Simulation parameter settings of the INS/SFBDSP-PPP tightly coupled system.

<table>
<thead>
<tr>
<th>Parameter settings of INS</th>
<th>Parameter settings of BDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant drift of gyros</td>
<td>White noise error of pseudo-range</td>
</tr>
<tr>
<td>White noise error of gyros</td>
<td>20 m</td>
</tr>
<tr>
<td>First-order Markov drift error of gyros</td>
<td>White noise error of pseudo-range</td>
</tr>
<tr>
<td>First-order Markov correlation time of gyros</td>
<td>0.02 m/s</td>
</tr>
<tr>
<td>First-order Markov bias of accelerometers</td>
<td>Equivalent distance error of C/A</td>
</tr>
<tr>
<td>First-order Markov correlation time of accelerators</td>
<td>0.5 m</td>
</tr>
</tbody>
</table>

In the INS/SFPPP-BDS tightly coupled system, the accuracy of the attitude and velocity measurements is also improved because of the high-precision observation. However, in the INS/SFPPP-BDS system, the pseudo-range and half-sum observations directly...
observe the positioning results, so the increase in the accuracy of the attitude and velocity is limited. The attitude and velocity errors of the two systems are compared in Tables 3 and 4.

**Simulation and analysis of the robust INS/SFPPP-BDS system with an adaptive cycle slip compensation model**

Although the introduction of the carrier phase observation into the INS/SFPPP-BDS system can significantly improve the positioning precision, the problem of cycle slip in the carrier phase cannot be neglected. As mentioned, the cycle slip causes the accuracy to decrease and destabilizes the INS/SFPPP-BDS system. To verify the proposed robust INS/SFPPP-BDS tightly coupled navigation algorithm with the adaptive cycle slip compensation model, which aims at decreasing the effect of the cycle slip, a dynamic simulation has been conducted. Based on the INS/SFPPP-BDS tightly coupled system simulation in the last section, a single cycle slip is injected into the carrier phase measurements of a visible satellite, as shown in Table 5.

The proposed robust algorithm with the adaptive cycle slip compensation model is applied to the INS/SFPPP-BDS system, and the simulation results are shown in Figures 5 and 6.

The curve of the carrier phase observations with cycle slip and the pseudo-range observations of the partially visible satellites is shown in Figure 5. The red solid line is the carrier phase observation, which has a cycle slip from 1000 s on Sv3, and the equivalent error of this cycle slip is approximately 50 m; the blue dash line is the pseudo-range observation, which is unacted on the cycle slip. Figure 6 shows the curve of the test statistics of the cycle slip detection in the robust INS/SFPPP-BDS tightly coupled system. The test statistics show that the curve has an abrupt change at...

---

**Table 2.** Comparison of the positioning error RMS values of INS/SFPPP-BDS and INS/BDS.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Longitude (m)</th>
<th>Latitude (m)</th>
<th>Height (m)</th>
<th>Longitude (m)</th>
<th>Latitude (m)</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INS/BDS</td>
<td>2.6643</td>
<td>3.6216</td>
<td>3.2520</td>
<td>0.6691</td>
<td>0.6212</td>
<td>1.0169</td>
</tr>
<tr>
<td>INS/SFPPP-BDS</td>
<td>3.2520</td>
<td>0.6691</td>
<td>1.0169</td>
<td>0.6212</td>
<td>0.6691</td>
<td>3.2520</td>
</tr>
</tbody>
</table>

**Table 3.** Comparison of the attitude error RMS values of INS/SFPPP-BDS and INS/BDS.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Roll (m)</th>
<th>Pitch (m)</th>
<th>Azimuth (m)</th>
<th>Roll (m)</th>
<th>Pitch (m)</th>
<th>Azimuth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INS/BDS</td>
<td>0.0937</td>
<td>0.0933</td>
<td>0.1686</td>
<td>0.0937</td>
<td>0.0827</td>
<td>0.1523</td>
</tr>
<tr>
<td>INS/SFPPP-BDS</td>
<td>0.0933</td>
<td>0.0827</td>
<td>0.1523</td>
<td>0.0937</td>
<td>0.0827</td>
<td>0.1523</td>
</tr>
</tbody>
</table>

**Table 4.** Comparison of the velocity error RMS values of INS/SFPPP-BDS and INS/BDS.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>East (m)</th>
<th>North (m)</th>
<th>Up (m)</th>
<th>East (m)</th>
<th>North (m)</th>
<th>Up (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INS/BDS</td>
<td>0.7200</td>
<td>0.9111</td>
<td>0.8699</td>
<td>0.5175</td>
<td>0.3384</td>
<td>0.5104</td>
</tr>
<tr>
<td>INS/SFPPP-BDS</td>
<td>0.7200</td>
<td>0.9111</td>
<td>0.8699</td>
<td>0.5175</td>
<td>0.3384</td>
<td>0.5104</td>
</tr>
</tbody>
</table>

**Table 5.** Simulation settings of cycle slip.

<table>
<thead>
<tr>
<th>Cycle slip</th>
<th>Satellite number</th>
<th>Equivalent observation error of cycle slip</th>
<th>Epoch at which cycle slip occurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sv3</td>
<td>50 m</td>
<td>1000 s</td>
<td></td>
</tr>
</tbody>
</table>

---

*Figure 4.* Attitude error comparison of INS/BDS and INS/SFPPP-BDS.

*Figure 5. Observation errors of the visible satellites of the robust INS/SFPPP-BDS tightly coupled system with cycle slip.*
1000 s, which indicates that the cycle slip is correctly detected in real time.

The key point of compensating the cycle slip is that the cycle slip estimation should be consistent with the change innovation in the system filter, as shown in Cycle slip compensation method and Robust INS/SFBDS-PPP integrated method sections. When no cycle slip occurs, the innovation of the system filter follows white noise, and its variance is consistent with the observation variance. When cycle slip occurs, the innovation has an abrupt change, which should be compensated in time to avoid a huge positioning error and even the divergence of the system filter. The cycle slip estimation can resolve this problem. The innovation change and cycle slip estimation with the proposed robust integrated algorithm are compared in Figures 7 and 8.

The innovation of INS/BDS and the robust INS/SFBDS-PPP under identical cycle slips as shown in Figure 5 are given in Figure 7, and the corresponding cycle slip estimation curve is shown in Figure 8. The estimation of the cycle slip correctly follows the change of the innovation. Before 1000 s, the estimation should be approximately zero because no cycle slip has occurred; at 1000 s, the estimation value has an abrupt change, which is consistent with the innovation change in Figure 7. To dynamically compensate the innovation error, the introduction of this estimation as a system state into the INS/SFPPP-BDS filter will improve the navigation precision. In Figure 7, the red solid line is the compensated innovation of INS/SFPPP-BDS, and the blue dash line is the uncompensated innovation of the system. It shows that the compensation of the innovation is effectively.

Under the above conditions, the corresponding navigation results of the INS/SFPPP-BDS system with the proposed robust fusion algorithm and classic Kalman filter are given in Figures 9 to 11.
In Figures 9 to 11, the red solid line denotes the navigation results of the proposed robust INS/SFPPP-BDS, and the blue dash line denotes INS/SFPPP-BDS with the Kalman filter. Under the condition of cycle slip in Figure 5, the positioning error is seriously affected. The cycle slip occurs to Sv3 starting from 1000s, which introduces approximately 50m error into the observation. Figure 9 shows that the positioning error of the blue dash line significantly increases from 1000s, whereas the red line has a stable positioning accuracy, and the positioning error is approximately 10m during the entire navigation, even when the cycle slip occurs. Thus, the positioning precision is significantly improved, particularly during the stage when cycle slips occur, because the proposed robust adaptive algorithm uses the optimal estimation of the cycle slip to repair the cycle slip in the observation, and in the proposed algorithm, the parameter of the filter can be adjusted with the change in the cycle slip. Particularly after 1000s, the positioning error of INS/SFPPP-BDS with the proposed algorithm is dramatically smaller than that with the Kalman filter. The RMS value of the positioning error is shown in Table 6. The comparison of the two results shows that the positioning accuracy can be significantly improved using the proposed algorithm.

The RMS values of the attitude and velocity errors of the INS/SFPPP-BDS system are shown in Tables 7 and 8. Under the effect of the robust navigation algorithm, the precisions of the attitude and velocity measurements are also improved.

![Figure 10. Velocity error comparison of INS/SFPPP-BDS with the robust fusion algorithm and Kalman filter.](image)

![Figure 11. Attitude error comparison of INS/SFPPP-BDS with the robust fusion algorithm and Kalman filter.](image)

**Table 6. Positioning error of INS/SFPPP-BDS with the Kalman filter and the robust navigation algorithm.**

<table>
<thead>
<tr>
<th></th>
<th>Kalman filter</th>
<th>Robust navigation algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude</td>
<td>41.5450</td>
<td>2.1241</td>
</tr>
<tr>
<td>Latitude</td>
<td>46.9047</td>
<td>2.3502</td>
</tr>
<tr>
<td>Height</td>
<td>103.5568</td>
<td>3.4189</td>
</tr>
</tbody>
</table>

**Table 7. Attitude error of INS/SFPPP-BDS with the Kalman filter and the robust navigation algorithm.**

<table>
<thead>
<tr>
<th></th>
<th>Kalman filter</th>
<th>Robust navigation algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>0.2633</td>
<td>0.0810</td>
</tr>
<tr>
<td>Pitch</td>
<td>0.2290</td>
<td>0.1147</td>
</tr>
<tr>
<td>Azimuth</td>
<td>0.1042</td>
<td>0.0597</td>
</tr>
</tbody>
</table>

**Table 8. Velocity error of INS/SFPPP-BDS with the Kalman filter and the robust navigation algorithm.**

<table>
<thead>
<tr>
<th></th>
<th>Kalman filter</th>
<th>Robust navigation algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>4.9195</td>
<td>4.195</td>
</tr>
<tr>
<td>North</td>
<td>5.3873</td>
<td>5.173</td>
</tr>
<tr>
<td>Up</td>
<td>11.7443</td>
<td>10.710</td>
</tr>
<tr>
<td>RMS (m/s)</td>
<td>0.7435</td>
<td>0.6902</td>
</tr>
</tbody>
</table>
Conclusion

Because the positioning precision of the traditional INS/BDS tightly coupled integrated navigation system cannot satisfy the increasing requirement of high precision, a new scheme of the INS/SFPPP-BDS tightly coupled integrated navigation system and a robust adaptive filtering algorithm that can adaptively compensate the cycle slip are proposed in this paper. The pseudo-range and half-sum observation models are built, and the proposed robust INS/SFPPP-BDS tightly coupled navigation algorithm with the adaptive cycle slip compensation model is presented. The INS/SFPPP-BDS uses a single-frequency receiver to integrate with the INS and subsequently uses the IGS products to measure the pseudo-range and carrier phase to improve the observation accuracy. In the adaptive cycle slip compensation model, the error model of the cycle slip is introduced into the filter as a state to improve the cycle slip estimation accuracy; additionally, the optimal estimation of the cycle slip will help to adjust the filtering parameter to avoid filtering divergence. The innovation of the filter is used as a test statistic of the cycle slip detection to detect and repair the cycle slip in real time. The simulation results show that the performance of the cycle slip estimation relates to the amplitude and duration of the cycle slip. The simulation of the INS/SFPPP-BDS system with cycle slips shows that the proposed algorithm can significantly improve the positioning accuracy, and the accuracy of the attitude and velocity can also be improved.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was partially supported by the Aeronautic Science Foundation of China (grant no. 20165552043), Science and Technology on Avionics Integration Laboratory.

ORCID iD

Rong Wang http://orcid.org/0000-0002-5965-6958

References


